# CONDITIONS FOR A SPACETIME TO BE FLRW

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## INTRODUCTION

Friedmann (1922, 1924), Lemaitre (1927, 1931), Robertson (1929, 1933) and Walker (1935) presented models known as FLRW models, which describe the large scale properties of our universe. Every FLRW spacetime is homogeneous and isotropic, i.e. it has a 6-dimensional isotropy group with 3-dimensional spacelike orbits. The groups are direct products of the form $B_3 \otimes O(3)$, where $B_3$ be one of the Bianchi groups. Grishchuk (1967) has investigated that only the Bianchi types I, V, $VIII_0$, $VII_h$ and IX are compatible with spherical symmetry.

## A SET OF NEW CONDITIONS FOR A SPACETIME TO BE FLRW

The following set of features is a necessary and sufficient condition for a spacetime to be FLRW.

(i) The metric obeys the Einstein field equations with a perfect fluid source.

(ii) The velocity field of the perfect fluid source has zero rotation, shear and acceleration.

Ellis (1971) has given the proof of their sufficiency from the equations of evolution of $\omega_{\alpha\beta}, \sigma_{\alpha\beta}$ and $\dot{u}^{\alpha}$. Those equations imply that a perfect fluid solution with $\sigma = \omega = 0 = \dot{u}^{\alpha}$ must be conformally flat. Stephani (1967) has given all conformally flat perfect fluid solutions. In general $\dot{u}^{\alpha} \neq 0$ but $\sigma = \omega = 0$. Krasinski (1981) has specialised them to the case $\dot{u}^{\alpha} = 0$ to obtain FLRW models. It is to be noted that the perfect fluid source is essential. The are solutions for which $\omega = \sigma = 0 = \dot{u}^{\alpha}$ but they are not FLRW because the source is not perfect fluid.

There is another invariant definition of FLRW spacetimes without the use of field equations. The following set of features is a necessary and sufficient condition for a spacetime to be FLRW.

(iii) The spacetime admits a foliation into spacelike hypersurfaces of constant curvature.

(iv) The congruence of lines Orthogonal to the leaves of the foliation are shearefree geodesics.

(v) The expansion scalar of the geodesic congruence has its gradient tangent to the geodesics.

There are some criteria for a FLRW limit. The necessary conditions are:

(i) The source must be a perfect fluid. In the case if the energy momentum tensor has more components such heat flow, then the additional quantities must be taken to zero. It is to be noted that a pure scalar field source cont dust are special cases of perfect fluid and are compatible with the FLRW spacetimes. Pure null field and pure electromagnetic are not compatible with FLRW geometry. Solutions with tachyon fluid for which the energy momentum tensor has the form

$$T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta - pg_{\alpha\beta}, \quad (1)$$

but the vector field $\underset{\sim}{u}$ is spacelike, have no FLRW limit.

(ii) $\dot{u}^\alpha = 0$ i.e. the acceleration must be zero. In comoving coordinates in which $u^\alpha = \delta_0^\alpha$ and $g_{oi} = 0, \quad i = 1, 2, 3,$ this condition has the simple form

$$g_{00,i} = 0, \quad i = 1, 2, 3 \quad (2)$$

(iii) The rotation must be zero i.e. $\omega = 0.$

(iv) The shear must be zero i.e. $\sigma = 0$ or

$$u^0 g_{ij,0} = \frac{2}{3}\theta g_{ij,} \qquad u, j = 1, 2, 3. \quad (3)$$

(v) The gradient of pressure must be colinear with the velocity field i.e.

$$u_{[\alpha} p,_{\beta]} = 0. \quad (4)$$

For a solution with pure perfect source, the eq. (4) is equivalent to $\dot{u}^\alpha = 0.$

(vi) The gradients of matter density and the expansion scalar must be colinear with velocity.

(vii) The barotropic equation of state

$$\rho,_{[\alpha} p,_{p]} = 0, \quad (5)$$

must hold.

(viii) The Weyl tensor must vanish.

(ix) The hypersurfaces orthogonal to the velocity field must have constant curvature.

## CONCLUDING REMARKS

We have presented a set of properties which is necessary and sufficient condition for a spacetime to be FLRW. The limit $\sigma = 0$ imply $\theta = 0$ (a static solution) or $\rho = 0$ (a vacuum solution or $\rho + p = 0,$ then with a pure perfect fluid source we get $\rho = -p =$ constant and the spacetime is a vacuum with cosmological constant. In such cases, no FLRW limit exists.

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$$C \neq 0$$

$\dot{u}^{\alpha}$.