

HORIZONS : THERMODYNAMICS

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ABSTRACT

General theory of relativity along with Einstein field equations and proper metric with signature $(-+++)$ i.e. plus two signature with units $G = \hbar = c = 1$. The Latin indices cover 0,1,2,3 while the Greek indices cover 1,2,3. An effective spacetime manifold obtained by removing the region blocked by the horizon. When the horizon is static, this effective manifold will have a nontrivial topology and leads to the association of a temperature with the horizon. It is obvious that there is a strong link between the kinematical aspects and the dynamics of gravity because of the structure of classical general relativity. The Euclidean sector incorporated the new ingredients which come when special relativity is combined to quantum mechanics and using the fact that when quantum theory is formulated in the Euclidean sector, a unique structure comes in the presence of horizons.

Keywords: thermodynamics, spacetime, Einstein-Hilbert, Einstein-Maxwell, Kerr-Newman metrics, Einstein-Hilbert, semiclassical, generalization, black hole mechanics.

INTRODUCTION

The Schwarzschild solution is the simplest solution to Einstein's field equations in general relativity showing a singular behaviour when presented in the most natural coordinate system, making the symmetries of the solution obvious. One of the metric coefficients g_{tt} vanishes on a surface \mathcal{H} of finite area while another g_{rr} diverges on the same surface. It was taken that these singularities

are due to bad choice of coordinates. But surface \mathcal{H} brought in new physical properties which have involved physicists very active in the field for decades. In the 1970s several studies presented that the Schwarzschild solution and its generalisations with horizons have strange relationship with laws of thermodynamics. The description of classical aspects of black hole thermodynamics may be obtained in (Bardeen, Carter and Hawking 1973), (DeWitt and DeWitt 1972), (Thorne, Price and Macdonald 1986). The work of (Bekenstein 1972, 1973, 1974) moved

these views forward and one was initially led to a system with entropy but no temperature. (Hawking 1975) resolved this paradox with particle creation by black holes, and it was very soon realised that there is a close relation between horizons and temperature as shown by (Fulling, 1973), (Davies 1975), (Gerlach 1976). Later study over four decades has reobtained these results and presented them in many different directions but unfortunately without any further insight. It may be very fair to say that the deep connection between thermodynamics, quantum theory and general relativity, which was hoped, is still elusive in the conventional methods. We are interested on certain specific aspects of thermodynamics of horizons and to explain a deeper connection between thermodynamics of horizons and gravity.

Classically only limited portions of the spacetime because of the horizons provide two effects when horizon is static:

- (i) Through Euclidean version of the quantum field theory it is clear that an effective spacetime manifold obtained by removing the region blocked by the horizon. When the horizon is static, this effective manifold will have a nontrivial topology and leads to the association of a temperature with the horizon. This comes in picture because the quantum theory contains information which classical theory does not, due to nonzero correlation functions on a spacelike hyper surface across the horizon.
- (ii) The gravitational action functional, when described in terms of the variables the family of observers may access, will have a boundary term proportional to the horizon area. Which is equivalent to associate a constant entropy per unit area of any horizon. It is also possible to obtain the Einstein-Hilbert action using the structure of the boundary term. This clarifies a peculiar connection between the boundary and surface terms of the Einstein-Hilbert action.

FORMULAE & COMPUTATIONAL METHOD:

There has been considerable amount of work in recent years in the latter approaches, such as by (Rovelli 1998), (Das and Mathur 2000), (Thiemann 2001), (Cardoso et al 2004), (Berti et al 2003) (Padmanabhan 2004). Most of the results obtained by these approaches are model dependent. On the other hand, since any viable microscopic model for quantum gravity reduces to Einstein gravity in the long wavelength limit, it is possible to obtain general results in the semiclassical limit of the theory which are independent of the microscopic details. While such a microscopic description is worth pursuing.

The features of stationary, 4-dimensional black holes have been well understood for quite some time. For example, in Einstein-Maxwell theory the situation is very simple. It is known that there is a unique 4-parameter family of stationary solution and, furthermore, these solutions are known explicitly, in closed form, given by the Kerr-Newman metrics and associated Maxwell fields by (theusler 1996)-(Black Hole Uniqueness Theorems, Cambridge University Press, Cambridge, England). Large families of stationary but distorted black holes are also known as presented by (Mysak and Szekeres 1966), (Geroch and Hatle 1982), (Fairhurst and Krishnan 2001). Finally, a framework has recently been presented to probe features of black holes by (Ashtekar et al 2000), (Ashtekar et al 2002). This isolated horizon framework enables one to assign mass and angular momentum to black holes in terms of values of the fields on the horizon itself, without any reference to infinity, and has also led to a generalisation of the zeroth and first laws of black hole mechanics by (Ashtekar, Fairhurst, and Krishnan 2002), (Ashtekar, Beetle and Lewandowski 2001).

However, black holes are rarely in equilibrium. For such fully dynamical black holes, there has been only one major result in exact general relativity, which is area theorem proved by Hawking in the early 1970s as given by (Hawking 1972), (Hawking and Ellis 1972); if matter satisfies

the dominant energy condition, the area of the black hole event horizon can never decrease. This theorem is similar to the second law of thermodynamics. However, it is a qualitative result; it does not give an explicit formula for the amount by which the area increases in physical situations. Now, the first law of black hole mechanics.

(1)

$$\delta E = \frac{k}{8\pi G} \delta a + \Omega \delta J,$$

does not give the change in the area of an isolated horizon to that in the energy and angular momentum, as the black hole makes a transition from one equilibrium state to nearby one, suggesting that there may be a fully dynamical version of eq. (1), relating the change in the black hole area to the energy and angular momentum fluxes. The expression needs a precise notion of the flux of gravitational energy across the horizon. one requires the framework presented by Bondi, Sachs, Newman, Penrose and others to introduce a viable, gauge invariant expression to this flux as (*Bondi, Vander Burg and Metzner 1962*), (*Ashtekar and Streubel 1981*), (*Wald and Zoupas 2000*). There is no proper generalisation of this framework and no satisfactory, gauge invariant notion of gravitational energy flux beyond perturbation theory.

There are two general considerations to extend the first law to fully dynamical situations. Let us consider a stellar collapse leading to the formation of a black hole. At the end of the process one has a black hole and one hopes that the energy in the final black hole should equal the total matter plus gravitation energy that fell across the horizon. Hence, the total integrated flux across the horizon should be well defined.

The question is how to define the surface of the black hole. Let us discuss the connection between dynamical horizons and closely related notion of trapping horizons introduced by (*Hayward 1994*). A smooth three-dimensional, spacelike submanifold H in spacetime M is known as a

dynamical horizon such that it may be foliated by a family of closed 2-surfaces, on each leaf, the expansion $\theta_{(1)}$ of one null normal l^a vanishes and the expansion $\theta_{(n)}$ of the other null normal n^a is strictly negative. Hence, a dynamical horizon H is a spacelike 3-manifold foliated by closed, marginally trapped 2-surfaces.

CONCLUSION

We have given review of research and development in the subject which has international and national status for the thermodynamics of horizons, dynamical horizons and their properties. Spacetime with horizons provide a resemblance to thermodynamic systems and it is possible to associate the notions of temperature and entropy to them. The approach will use two principles (i) the physical theories must be presented for each observer entirely in terms of variables any given observer may access, and (ii) consistent formulation of quantum field theory needs analytic continuation to the complex plane. When these two aspects applied together in spacetimes with horizons are powerful to give several results in a unified manner.

It is obvious that there is a strong link between the kinematical aspects and the dynamics of gravity because of the structure of classical general relativity. the Euclidean sector incorporated the new ingredients which come when special relativity is combined to quantum mechanics and using the fact that when quantum theory is formulated in the Euclidean sector, a unique structure comes in the presence of horizons.

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