MULTIDIMENSIONAL OPERATORS INVOLVING MULTIVARIABLE - A FUNCTION

Anil Ramawat,

Department of Mathematics, Government College, Sojat City, Rajasthan, India.

ABSTRACT

In the present paper, we study certain multidimensional fractional integral operators involving a general A-function in their kernel. We give some basic properties of these operators, and then establish two theorems and two corollaries, which are believed to be new. These basic theorems exhibit structural relationships between the multidimensional integral transforms. The one-and –two –dimensional analogues of these results, which are new and of interest in themselves, can easily be deduced. Special cases of these latter theorems will give rise to certain known results obtained from time to time by several earlier authors.

INTRODUCTION

Gautam and Goyal [5] defined and represented the multivariable A-function as follows:

$$A[z_1,\ldots,z_r] = A^{m,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}$$

$$\cdot\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix} (a_j;A'_j,\ldots,A_j^{(r)})_{1,p};(c'_j,C'_j)_{1,p_1};\ldots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \\ (b_j;B'_j,\ldots,B_j^{(r)})_{1,q};(d'_j,D'_j)_{1,q_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,q_r} \end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\int_{L_r}\theta_1(s_1)\ldots\theta_r(s_r)\Phi(s_1,\ldots,s_r)z_1^{s_1}\ldots z_r^{s_r}\,.ds_1\ldots ds_r \qquad (1.1)$$

Where $\omega=\sqrt{-1}$;

$$\theta_i(s_i) = \frac{\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-D_j^{(i)}s_i)\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+C_j^{(i)}s_i)}{\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}+D_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-C_j^{(i)}s_i)}$$

$$\forall i\in\{1,\ldots,r\} \qquad (1.2)$$

$$\Phi(s_1,\ldots,s_r) = \frac{\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{i=1}^{r}A_j^{(i)}s_i)\prod_{j=1}^{m}\Gamma(b_j-\sum_{i=1}^{r}B_j^{(i)}s_i)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{i=1}^{r}A_j^{(i)}s_i)\prod_{j=m+1}^{q}\Gamma(1-b_j+\sum_{i=1}^{r}B_j^{(i)}s_i)} \qquad (1.3)$$

Here $m, n, p, q, m_i, n_i, p_i$, and $q_i\ (i = 1, \ldots, r)$ are non-negative integers and all $a_j^{'}s, b_j^{'}s, d_j^{(i)'}s, c_j^{(i)'}s, A_j^{(i)'}s$ and $B_j^{(i)'}s$ are complex numbers.

The multiple integral defining the A-function of r-variables converges absolutely if

$$|\arg(\Omega_i)z_k| < \frac{\pi}{2}\eta_i, \xi_i^* = 0, \eta_i > 0 \quad (1.4)$$

$$\Omega_i = \prod_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}} \prod_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}} \prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}} \cdot \prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}}$$

$$, \forall i \in \{1, \ldots, r\}; \quad (1.5)$$

$$\xi^*{}_i = I_m\left[\sum_{j=1}^{p} A_j^{(i)} - \sum_{j=1}^{q} B_j^{(i)} + \sum_{j=1}^{q_i} D_j^{(i)} - \sum_{j=1}^{q_i} C_j^{(i)}\right], \ \forall i \in \{1, \ldots, r\} \quad (1.6)$$

$$\eta_i = \mathrm{Re}\left[\sum_{j=1}^{n} A_j^{(i)} - \sum_{j=n+1}^{p} A_j^{(i)} + \sum_{j=1}^{m} B_j^{(i)} - \sum_{j=m+1}^{q} B_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\right]$$

$$\forall i \in \{1, \ldots, r\}; \quad (1.7)$$

If we take $A_j^{'}s, B_j^{'}s, C_j^{'}s$ and $D_j^{'}s$ as real and positive and $m = 0$, the A-function reduces to multivariable H-function of Srivastava and Panda [17]

We are using the multivariable A-function in the following concise form throughout the text.

$$A[z_1,\ldots,z_r] = A_{p,q:N_r}^{m,n:M_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} P : P_r^{(r)} \\ Q : Q_r^{(r)} \end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\int_{L_r} \theta_1(s_1)\ldots\theta_r(s_r)\Phi(s_1,\ldots,s_r) z_1^{s_1}\ldots z_r^{s_r}\ .ds_1\ldots ds_r \quad (1.8)$$

Where $\omega = \sqrt{-1}$;

$M_r = m_1, n_1; \ldots; m_r, n_r;$

$N_r = p_1, q_1; \ldots; p_r, q_r;$

$P = (a_j; A_j^{'}, \ldots, A_j^{r})_{1,p};$ $\qquad Q = (b_j; B_j^{'}, \ldots, B_j^{r})_{1,q};$

$P_r^{(r)} = (c_j^{'}, C_j^{'})_{1,p_1}; \ldots; (c_j^{(r)}, C_j^{(r)})_{1,p_r};$

And the definition of the functions $\theta_i(s_i)$ $i = 1, \ldots, r$; $\Phi(s_1,\ldots,s_r)$ and the condition of existence of the multivariable A-function are the same as mentioned by Gautam and Goyal [5].

We introduce the fractional integration operators by means of the following integral equations:

$$Y\{f(x)\} = Y\{f(x); t_1,\ldots,t_r; \gamma_1,\ldots,\gamma_r\} = \prod_{i=1}^{r}\left(t_i^{\gamma_i - 1}\right)$$

$$\int_0^{t_1}...\int_0^{t_r}\prod_{i=1}^{r}\left\{(x_i)^{\gamma_i}\phi\left(\frac{x_1}{t_1},...,\frac{x_r}{t_r}\right)\right\}f(x)dx_1...dx_r \quad (1.9)$$

And

$$N\{f(x)\}=N\{f(x);t_1,...,t_r;\delta_1,...,\delta_r\}=\prod_{i=1}^{r}\left(t_i^{\delta_i}\right)$$

$$\int_{t_1}^{\infty}...\int_{t_r}^{\infty}\prod_{i=1}^{r}\left\{(x_i)^{-\delta_i-1}\phi\left(\frac{t_1}{x_1},...,\frac{t_r}{x_r}\right)\right\}f(x)dx_1...dx_r \quad (1.10)$$

Where the kernel ϕ is such that the above integrals make sense. The above operators exist under the following conditions:

(i) $p_i \le 1, q_i < \infty, \frac{1}{p_i}+\frac{1}{q_i}=1, i=1,2,...,r$

(ii) $\mathrm{Re}(\gamma_i) > -\frac{1}{q_i}; \mathrm{Re}(\delta_i) > -\frac{1}{p_i}; i=1,2,...,r$

(iii) $f(x)\in L_{p_i}((0,\infty),...,(0,\infty)); i=1,2,...,r$

(iv) $0\le m_i \le q_i, 0 \le n_i \le p_i, q_k \ge 0, 0 \le n_k \le p_k$

The following special case of the multidimensional fractional integral operators involving product of Gauss's hypergeometric functions ([14],p.153,eq. (i) and (ii))will be used in Section 3.

$$I\{f(x)\}=\prod_{i=1}^{r}\left\{\frac{t_i^{\gamma_i-1}}{\Gamma(1-a_i)}\right\}\int_0^{t_1}...\int_0^{t_r}\prod_{i=1}^{r}\left\{(x_i)^{\gamma_i}\,{}_2F_1\left(\alpha_i,\beta_i+m;\beta_i;\frac{x_i}{t_i}\right)\right\}f(x)dx_1...dx_r \quad (1.11)$$

And

$$K\{f(x)\}=\prod_{i=1}^{r}\left\{\frac{t_i^{\delta_i}}{\Gamma(1-\alpha_i)}\right\}\int_{t_1}^{\infty}...\int_{t_r}^{\infty}\prod_{i=1}^{r}\left\{(x_i)^{-\delta_i-1}\,{}_2F_1\left(\alpha_i,\beta_i+m;\beta_i;\frac{t_i}{x_i}\right)\right\}f(x)dx_1...dx_r \quad (1.12)$$

The conditions of existence of these operators follow easily from the conditions given in the paper referred to above.

The generalized multidimensional integral transform T, defined below, will also be required during the course of our study:

$$T\{f(x);s_1,...,s_r\}=\int_0^{\infty}...\int_0^{\infty}k(s_1x_1,...,s_rx_r)\,f(x)dx_1...dx_r \quad (1.13)$$

Where $k(s_1x_1,...,s_rx_r)$ is the kernel of the transform T and the multiple integral occurring in the equation (1.8) is assumed to be convergent.

The following multivariable A-function transform will also be used in the sequel:

$$A\{f(x);s_1,...,s_r\}=\int_0^\infty ...\int_0^\infty A^{m,n:m_1,n_1;...;m_r,n_r}_{p,q:p_1,q_1;...;p_r,q_r}$$

$$\cdot\left[\begin{matrix} s_1x_1 \\ \vdots \\ s_rx_r \end{matrix}\left|\begin{matrix}(a_j;A_j^{'},...,A_j^{(r)})_{1,p};(c_j^{'},C_j^{'})_{1,p_1};...;(c_j^{(r)},C_j^{(r)})_{1,p_r} \\ (b_j;B_j^{'},...,B_j^{(r)})_{1,q};(d_j^{'},D_j^{'})_{1,q_1};...;(d_j^{(r)},D_j^{(r)})_{1,q_r}\end{matrix}\right.\right] f(x)dx_1...dx_r \quad (1.14)$$

SOME BASIC PROPERTIES

Property 1. If $f(x)\in L_{p_i}((0,\infty),...,(0,\infty));1\le p_i\le 2$ (or $f(x)\in M_{p_i}((0,\infty),...,(0,\infty));\ p_i>2$,where $M_{p_i}((0,\infty),...,(0,\infty))$ denotes the class of all functions $f(x)\in L_{p_i}((0,\infty),...,(0,\infty));\ p_i>2$, which are the inverse Mellin transforms of functions belonging to $L_{q_i}((-\infty,\infty),...,(-\infty,\infty));\mathrm{Re}(\gamma_i)>\frac{1}{q_i},\mathrm{Re}(\delta_i)>\frac{1}{p_i},\frac{1}{p}+\frac{1}{q}=1(i=1,2,...,r)$ and the multidimensional Mellin transform of the function $f(x)$ exists, then

(a) $M\left[Y\{f(x);t_1,...,t_r;\gamma_1,...,\gamma_r\};s_1,...,s_r\right]=$

$$M\left[f(x);s_1,...,s_r\right]N\{1;t_1,...,t_r;\gamma_1-s_1+1,...,\gamma_r-s_r+1\} \quad (2.1)$$

(b) $M\left[N\{f(x);t_1,...,t_r;\delta_1,...,\delta_r\};s_1,...,s_r\right]=$

$$M\left[f(x);s_1,...,s_r\right]Y\{1;t_1,...,t_r;\delta_1+s_1+1,...,\delta_r+s_r+1\} \quad (2.2)$$

Where the symbol M occurring in (2.1) and (2.2) stands for the multidimensional Mellin transform defined in the following way:

$$M\{f(x);s_1,...,s_r\}=\int_0^\infty ...\int_0^\infty f(x)\prod_{i=1}^{r}\left(x_i^{s_i-1}\right)dx_1...dx_r \quad (2.3)$$

Provided that the multiple integral involved in (2.3) exists.

Proof: To prove (2.1), we use (2.3)and (1.9) to obtain

$$M\left[Y\{f(x)\}\right]=\int_0^\infty ...\int_0^\infty (t_i^{s_i-1})\left\{\int_0^{t_1}...\int_0^{t_r}\prod_{i=1}^{r}\left(x_i^{\gamma_i}\right)f(x)\ \phi\left(\frac{x_1}{t_1},...,\frac{x_r}{t_r}\right)\right\}dt_1...dt_r \quad (2.4)$$

Now interchanging the orders of t_i and $x_i(i=1,2,...,r)$ integrals in (2.4), which is easily seen to be permissible under the conditions stated with (2.1) and interpreting the results thus obtained with the help of (1.10), we arrive at the required result (2.1).

The result (2.2) can be established in a similar manner.

Property 2. If $f(x)\in L_{p_i}((0,\infty),...,(0,\infty));1\le p_i\le 2$, $g(x)\in L_{q_i}((0,\infty),...,(0,\infty))$; ,

$\mathrm{Re}(\gamma_i)>\max\left(-\frac{1}{p_i},-\frac{1}{q_i}\right),(i=1,2,...,r)$, then

$$\int_0^\infty ...\int_0^\infty f(x)Y\{g(x)\}dx_1...dx_r=\int_0^\infty ...\int_0^\infty g(x)N\{f(x)\}dx_1...dx_r \quad (2.5)$$

Provided that the multiple integrals involved in (2.5) are absolutely convergent.

Property 3.

(a) $Y\{f(wx);t_1,...,t_r;\gamma_1,...,\gamma_r\}=Y\{f(x);t_1w_1,...,t_rw_r;\gamma_1,...,\gamma_r\}$ (2.6)

(b) $N\{f(wx);t_1,...,t_r;\delta_1,...,\delta_r\}=W\{f(x);t_1w_1,...,t_rw_r;\delta_1,...,\delta_r\}$ (2.7)

Provided that the multiple integrals involved in (2.6) and (2.7) are absolutely convergent.

Property 4.

(a) $Y\{f(1/x);t_1,...,t_r;\gamma_1,...,\gamma_r\}=Y\{f(x);1/t_1,...,1/t_r;\gamma_1+1,...,\gamma_r+1\}$ (2.8)

(b) $N\{f(1/x);t_1,...,t_r;\delta_1,...,\delta_r\}=W\{f(x);1/t_1,...,1/t_r;\delta_1-1,...,\delta_r-1\}$ (2.9)

Provided that the multiple integrals involved in (2.8) and (2.9) are absolutely convergent.

RELATIONSHIP BETWEEN MULTIDIMENSIONAL FRACTIONAL INTEGRAL OPERATORS AND MULTIDIMENSIONAL INTEGRAL TRANSFORMS

In this section, we shall establish two most general theorems exhibiting interconnections between the fractional integral operators Y and N defined by (1.9) and (1.10) respectively and the integral transform T defined by (1.13). Next, we give two interesting corollaries interconnecting the multidimensional fractional integral operators defined by (1.1) and (1.12) and the multidimensional A-function transform defined by (1.14).

Theorem 1. If

$$\tau(s_1,..,s_r)=T\{\psi(u^\rho)g(u);s_1,...,s_r\}=$$

$$\int_0^\infty ...\int_0^\infty k(su)\psi(u^\rho)g(u)du_1...du_r \quad (3.1)$$

And

$$\psi(t_1,...,t_r)=Y\{f(x^\sigma);t_1,...,t_r\}=$$

$$\prod_{i=1}^r\left(x_i^{-\gamma_i-1}\right)\int_0^{t_1}...\int_0^{t_r}f(x^\sigma)\prod_{i=1}^r\left(x_i^{\gamma_i}\right)\phi\left(\frac{x_1}{t_1},...,\frac{x_r}{t_r}\right)\Bigg\}dx_1...dx_r \quad (3.2)$$

Then

$$\tau(s_1,..,s_r)=\frac{1}{\rho_1..\rho_r}\int_0^\infty...\int_0^\infty f(x^\sigma)$$

$$\theta(s_1,...,s_r,x_1,...,x_r,\rho_1,...,\rho_r)dx_1...dx_r \quad (3.3)$$

Where

$$\theta(s_1,...,s_r,x_1,...,x_r,\rho_1,...,\rho_r)=$$

$$Y\left\{\prod_{i=1}^{r}\left(x_i^{\frac{1}{\rho_i}-1}\right)g\left(x^{\frac{1}{\rho}}\right)k\left(sx^{\frac{1}{\rho}}\right);t_1,...,t_r\right\}=$$

$$\prod_{i=1}^{r}\left(t_i^{\gamma_i}\right)\int_{t_1}^{\infty}...\int_{t_r}^{\infty}\prod_{i=1}^{r}\left(x_i^{\frac{1}{\rho_i}-\gamma_i-2}\right)g\left(x^{\frac{1}{\rho}}\right)k\left(sx^{\frac{1}{\sigma}}\right)\ \phi\left(\frac{t_1}{x_1},...,\frac{t_r}{x_r}\right)\Bigg\}dx_1...dx_r \quad (3.4)$$

Where ρ_i and $\sigma_i(i=1,2,...,r)$ are non zero real numbers of the same sign and all the integrals involved in equations (3.1) to (3.4) are assumed to be absolutely convergent. Also in (3.4) $k\left(sx^{\frac{1}{\sigma}}\right)$ stands for $k\left(s_1x_1^{\frac{1}{\rho_1}},...,s_rx_r^{\frac{1}{\rho_r}}\right)$ and so on.

Proof: Applying the formula (2.5) to the pair of equations (3.2) and (3.4), we get

$$\int_0^\infty...\int_0^\infty f(x^\sigma)\theta(s_1,...,s_r,x_1,...,x_r,\rho_1,...,\rho_r)dx_1...dx_r=$$

$$\int_0^\infty...\int_0^\infty\prod_{i=1}^{r}\left(x_i^{\frac{1}{\rho_i}-1}\right)g\left(x^{\frac{1}{\rho}}\right)k\left(sx^{\frac{1}{\sigma}}\right)\psi(x)dx_1...dx_r \quad (3.5)$$

Now changing the variables of integrations on the right hand side of (3.5) slightly and interpreting the result thus obtained with the help of (3.1), we easily arrive at (3.3) after a little simplification.

If in the above theorem, we replace ρ_i and $h_i(i=1,2,...,r)$ and take

$$g(x)=\prod_{i=1}^{r}\left\{x_i^{h_i(c_i+1)-1}\right\},$$

$$\phi(x)=\prod_{i=1}^{r}\left\{\frac{1}{\Gamma(1-\alpha_i)}\,{}_2F_1(\alpha_i,\beta_i+m;\beta_i;x_i)\right\}$$

And T to be multidimensional A-function transform defined by (1.13), the right hand side of equation (3.4) assumes the following form:

$$\prod_{i=1}^{r}\left\{\frac{t_i^{\gamma_i}}{\Gamma(1-\alpha_i)}\right\}\int_{t_1}^{\infty}\cdots\int_{t_r}^{\infty}\prod_{i=1}^{r}\left\{x_i^{c_i-\gamma_i-1}\,{}_2F_1\left(\alpha_i,\beta_i+m;\beta;\frac{t_i}{x_i}\right)\right\}$$

$$I\left(s_1x_1^{\frac{1}{h_1}},\ldots,s_rx_r^{\frac{1}{h_r}}\right)dx_1\ldots dx_r \qquad (3.6)$$

On evaluation the above integral with the help of known results ([4],p.398,eq.(2)) and ([3],p.105,eq.(1)) and the definition of multivariable A-function (1.1) and substituting the values of $\theta(s_1,\ldots,s_r,x_1,\ldots,x_r,\rho_1,\ldots,\rho_r)$ thus obtained, in the right hand side of equation (3.3), we obtain the following interesting corollary with the help of equations (3.1) and (3.3).

Corollary 1. If $h_i>0,\mathrm{Re}(1-\alpha_i)>m,m\in W$,(the set of whole number) $\sigma_i(i=1,2,\ldots,r)$ are non-zero real numbers of the same sign.

$$\beta_i\neq 0,-1,-2,\ldots;\mathrm{Re}(C_i)\geq 0$$

$$f(x)=\frac{o\left(x_i^{A_i}\right), \text{ forsmallvaluesof } x_i}{o\left(x_i^{B_i}e^{-C_ix_i}\right) \text{ forl arg evaluesof } x_i};i=1,2,\ldots,r$$

The multivariable A-function satisfy the conditions corresponding appropriately to those given by Srivastava and Panda ([18],p.130, eqs. (1.1) and (1.15)), then

$$I\left\{\prod_{i=1}^{r}x_i^{h_i(c_i+1)-1}\psi(x^h);^{0,n_2:\ldots:0,n_r:(m',n'):\ldots:\left(m^{(r)},n^{(r)}\right),}_{p_2,q_2:\ldots:p_r,q_r:(p',q'):\ldots:\left(p^{(r)},q^{(r)}\right),}\right.$$

$$\begin{matrix}(a_{2j},\alpha_{2j}^{'},\alpha_{2j}^{''})_{1,p_2}:\ldots:(\alpha_{rj},\alpha_{rj}^{'},\ldots,\alpha_{rj}^{(r)})_{1,p_r}:(a_j^{'},\alpha_j^{'})_{1,p'-1}:\ldots:(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}-1}\\(b_{2j},\beta_{2j}^{'},\beta_{2j}^{''})_{1,q_2}:\ldots:(b_{rj},\beta_{rj}^{'},\ldots,\beta_{rj}^{(r)})_{1,q_r}:(b_j^{'},\beta_j^{'})_{1,q'-1}:\ldots:(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}-1}\end{matrix}$$

$$\left.\begin{matrix}\left(1-\alpha_1+\gamma_1-c_1,\frac{1}{h_1}\right),\ldots,\left(1-\alpha_r+\gamma_r-c_r,\frac{1}{h_r}\right)\\\left(1-\beta_1+\gamma_1-m-c_1,\frac{1}{h_1}\right),\ldots,\left(1-\beta_r+\gamma_r-m-c_r,\frac{1}{h_r}\right)\end{matrix};s_1,\ldots,s_r\right\}=$$

$$\frac{1}{\prod_{i=1}^{r}(\beta_i)_m}I\left\{\prod_{i=1}^{r}x_i^{h_i(c_i+1)-1}f(x^{h\sigma});^{0,n_2:\ldots:0,n_r:(m',n'):\ldots:\left(m^{(r)},n^{(r)}\right),}_{p_2,q_2:\ldots:p_r,q_r:(p',q'):\ldots:\left(p^{(r)},q^{(r)}\right),}\right.$$

$$\begin{matrix}(a_{2j},\alpha_{2j}^{'},\alpha_{2j}^{''})_{1,p_2}:\ldots:(\alpha_{rj},\alpha_{rj}^{'},\ldots,\alpha_{rj}^{(r)})_{1,p_r}:(a_j^{'},\alpha_j^{'})_{1,p'-1}:\ldots:(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}-1}\\(b_{2j},\beta_{2j}^{'},\beta_{2j}^{''})_{1,q_2}:\ldots:(b_{rj},\beta_{rj}^{'},\ldots,\beta_{rj}^{(r)})_{1,q_r}:(b_j^{'},\beta_j^{'})_{1,q'-1}:\ldots:(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}-1}\end{matrix}$$

$$\left.\begin{matrix}\left(\gamma_1-c_1,\frac{1}{h_1}\right),\ldots,\left(\gamma_r-c_r,\frac{1}{h_r}\right)\\\left(1-\beta_1+\gamma_1-c_1,\frac{1}{h_1}\right),\ldots,\left(1-\beta_r+\gamma_r-c_r,\frac{1}{h_r}\right)\end{matrix};s_1,\ldots,s_r\right\} \qquad (3.7)$$

Where

$$\psi(t_1,...,t_r) = I\{f(x);t_1,...,t_r\} = \prod_{i=1}^{r}\left\{\frac{t_i^{\gamma_i}}{\Gamma(1-\alpha_i)}\right\}$$

$$\int_0^{t_1}...\int_0^{t_r}\prod_{i=1}^{r}\left\{x_i^{\gamma_i}\,{}_2F_1\left(\alpha_i,\beta_i+m;\beta_i;\frac{x_i}{t_i}\right)\right\}f(x^\sigma)dx_1...dx_r \tag{3.8}$$

Provided that

$$\mathrm{Re}\left(c_i-\gamma_i+\frac{1}{h_i}\frac{b_j^{(i)}-1}{B_j^{(i)}}\right)<0;1\le j\le n_i$$

$$\mathrm{Re}\left(\sigma_i B_i+c_i+1+\frac{1}{h_i}\frac{b_j^{(i)}-1}{B_j^{(i)}}\right)<0;1\le j\le n_i$$

$$\mathrm{Re}\left(\sigma_i A_i+c_i+1+\frac{1}{h_i}\frac{d_k^{(i)}-1}{D_k^{(i)}}\right)<0;1\le k\le m_i$$

$$\mathrm{Re}\left(\sigma_i A_i+\gamma_i+1\right)>0;i=1,2,...,r$$

Theorem 2. If

$$\tau(s_1,..,s_r)=T\{\psi(u^\rho)g(u);s_1,...,s_r\}=$$

$$\int_0^\infty...\int_0^\infty k(su)\psi(u^\rho)g(u)du_1...du_r \tag{3.9}$$

And

$$\psi(t_1,...,t_r)=N\{f(x^\sigma);t_1,...,t_r\}=$$

$$\prod_{i=1}^{r}\left(t_i^{\gamma_i}\right)\int_{t_1}^\infty...\int_{t_r}^\infty f(x^\sigma)\prod_{i=1}^{r}\left(x_i^{-\gamma_i-1}\right)\phi\left(\frac{t_1}{x_1},...,\frac{t_r}{x_r}\right)\Bigg\}dx_1...dx_r \tag{3.10}$$

then

$$\tau(s_1,..,s_r)=\frac{1}{\rho_1...\rho_r}\int_0^\infty...\int_0^\infty f(x^\sigma)$$

$$\theta(s_1,...,s_r,t_1,...,t_r,\rho_1,...,\rho_r)dx_1...dx_r \tag{3.11}$$

Where

$$\theta(s_1,...,s_r,t_1,...,t_r,\rho_1,...,\rho_r)=$$

$$Y\left\{\prod_{i=1}^{r}\left(x_i^{\frac{1}{\rho_i}-1}\right)g\left(x^{\frac{1}{\rho}}\right)k\left(sx^{\frac{1}{\rho}}\right);t_1,\ldots,t_r\right\}=$$

$$\prod_{i=1}^{r}\left(t_i^{-\gamma_i-1}\right)\int_0^{t_1}\ldots\int_0^{t_r}\prod_{i=1}^{r}\left(x_i^{\frac{1}{\rho_i}+\gamma_i-1}\right)g\left(x^{\frac{1}{\rho}}\right)k\left(sx^{\frac{1}{\rho}}\right)\phi\left(\frac{x_1}{t_1},\ldots,\frac{x_r}{t_r}\right)dx_1\ldots dx_r \qquad (3.12)$$

Where ρ_i and $\sigma_i(i=1,2,\ldots,r)$ are non zero real numbers of the same sign and all the integrals involved in equations (3.1) to (3.4) are assumed to be absolutely convergent.

Proof: The proof of the above theorem can be easily developed on the lines similar to those of theorem 1.

Again, if in theorem 2, we replace ρ_i by $h_i(i=1,2,\ldots,r)$ and take

$$g(x)=\prod_{i=1}^{r}\left\{x_i^{h_i(c_i+1)-1}\right\}$$

$$\phi(x)=\prod_{i=1}^{r}\left\{\frac{1}{\Gamma(1-\alpha_i)}\,{}_2F_1\left(\alpha_i,\beta_i+m;\beta_i;x_i\right)\right\}$$

And T to be multidimensional A-function transform defined by (1.13), then proceeding on the lines similar to those of corollary 1.

REFERENCES

1. Erdelyi, A.; on fractional integration and its application to the theory of Hankel transforms, Quart. J. Math. Oxford Ser. (2)2, (1920), 293-303.
2. Erdelyi, A.; On some fractional transformations, Univ. Politec. Torino. Rend. Sem. Mat. 10, (1950), 217-234.
3. Erdelyi, A. et.al. Higher Transcendental Functions, vol. I, McGraw-Hill, New York (1953).
4. Erdelyi, A. et.al. Tables of Integral Transforms, vol. II, McGraw-Hill, New York (1954).
5. Gautam, G.P. and Goyal, A.N.; On Fourier kernels and asymptotic expansion. Indian J. Pure Appl. Math. 12, (1981), 1109-1105.
6. Gupta, K.C. , Goyal, S.P. and Handa, S.; Fractional integral operators involving two variables, Univ. Nac. Tucuman Rev. Ser. A 26, (1976), 159-171.
7. Handa, S.; A study of generalized functions of one and two variables, Indian J. of Math., (1976), 11-16.
8. Kober, H.; On fractional integrals and derivatives, Quart. J.Math. Oxford Ser. 2(II), (1940), 193-211.
9. Kalla, S.L.; Some theorems on fractional integration, Proc. Nat. Acad. Sci. India Sect. A 36, (1966), 1007-1012.
10. Kalla, S.L.; Some theorems on fractional integration II, Proc. Nat. Acad. Sci. India Sect. 39, (1969), 44-56.
11. Kalla, S.L.; Fractional integration operators involving generalized hypergeometric functions, Univ. Nac. Tucuman. Rev. Ser. A 20, (1970), 93-100.
12. Mathur, S.L.; Astudy of boundary value problems and special functions, Proc. Acad. Sci India Sect.A 43,(1974),56-61.

13. Saxena, R.K.; On fractional integral operators, Math. Zeitscher. 95, (1967), 288-291.
14. Saxena, R.K. and Modi, G.C.; Multidimensional fractional integration operators associated with hypergeometric kernels, Nat. Acad. Sci. Lett. 13,(1980),153-158.
15. Sneddon, I.N.; Mixed Boundary Value Problems in Potential Theory, North-Holland Publishing Co. , Amsterdam (1966).
16. Srivastava, H.M. and Buschmann R.G.; Composition of fractional integral operators involving Fox's H -function, Acta Mexicana Ci Tecn 7,(1973),21-28.
17. Srivastava, H.M. and Panda, R.; Some bilateral generating functions for a class of generalized polynomials, J. Reine Angew Math. 283/284, (1976)265-274.
18. Srivastava, H.M. and Panda, R.;Expansion theorems for the H -function of several complex variables, J.Reine Angew Math 288, (1976), 129-145.
19. Srivastava, H.M. and Panda, R.; Certain multidimensional integral transformations I and II, Nederl. Akad Wetensch Proc. Ser. A 81, (1978), 118-141.
20. Srivastava, H.M. , Gupta, K.C. and Goyal, S.P.; The H -function of One and Two Variables With Applications, South Asian Publishers Pvt. Ltd., New Delhi and Madras (1982).