ON SUMATION FORMULAE FOR THE MULTIVARIABLE HYPERGEOMETRIC FUNCTION

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ABSTRACT

In the present paper, the authors has been derived finite summation formulae for the multivariable -function. Since the multivariable -function includes a large number of special functions of one and more variables as its particular cases.

Keywords: Multivariable -function, Summation formula, Contiguous relations.

INTRODUCTION

The object of this paper is to establish four finite summation formulae for the multivariable I -function. These formulae will yield a number of new and known results.

The multivariable I -function introduced by Prasad [4] will be define and represent it in the following manner:

$$I\left[z_1,\ldots,z_r\right]=I^{0,n_2:\ldots:0,n_r:(m',n'):\ldots:\left(m^{(r)},n^{(r)}\right)}_{p_2,q_2:\ldots:p_r,q_r:(p',q'):\ldots:\left(p^{(r)},q^{(r)}\right)}\left[z_1,\ldots,z_r\left|\begin{matrix}(a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}}\\(b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}}\end{matrix}\right.\right]$$

$$=\frac{1}{(2\pi w)^r}\int_{L_1}\ldots\int_{L_r}\phi_1(s_1)\ldots\phi_r(s_r)\psi(s_1,\ldots,s_r)z_1^{s_1},\ldots,z_r^{s_r}\,ds_1\ldots ds_r \qquad (1.1)$$

Where

$$w=\sqrt{(-1)}$$

$$\phi_i(s_i)=\frac{\prod_{j=1}^{m^{(i)}}\Gamma\left(b_j^{(i)}-\beta_j^{(i)}s_i\right)\prod_{j=1}^{n^{(i)}}\Gamma\left(1-a_j^{(i)}+\alpha_j^{(i)}s_i\right)}{\prod_{j=m^{(i)}+1}^{q^{(i)}}\Gamma\left(1-b_j^{(i)}+\beta_j^{(i)}s_i\right)\prod_{j=n^{(i)}+1}^{p^{(i)}}\Gamma\left(a_j^{(i)}-\alpha_j^{(i)}s_i\right)}\quad\forall i\in(1,2,\ldots,r) \qquad ($$

$$\psi(s_1,\ldots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma\left(1-a_{2j}+\sum_{i=1}^{2}\alpha_{2j}^{(i)}s_i\right)\prod_{j=1}^{n_3}\Gamma\left(1-a_{3j}+\sum_{i=1}^{3}\alpha_{3j}^{(i)}s_i\right)}{\prod_{j=n_2+1}^{p_2}\Gamma\left(a_{2j}-\sum_{i=1}^{2}\alpha_{2j}^{(i)}s_i\right)\prod_{j=n_3+1}^{p_3}\Gamma\left(a_{3j}-\sum_{i=1}^{3}\alpha_{3j}^{(i)}s_i\right)}$$

$$\frac{\ldots\prod_{j=1}^{n_r}\Gamma\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i\right)}{\ldots\prod_{j=n_r+1}^{p_r}\Gamma\left(a_{rj}-\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i\right)\prod_{j=1}^{q_2}\Gamma\left(1-b_{2j}-\sum_{i=1}^{2}\beta_{2j}^{(i)}s_i\right)\ldots}\;\frac{1}{\prod_{j=1}^{q_r}\Gamma\left(1-b_{rj}-\sum_{i=1}^{r}\beta_{rj}^{(i)}s_i\right)} \quad (1.3)$$

$\alpha_j^{(i)},\beta_j^{(i)},\alpha_{kj}^{(i)},\beta_{kj}^{(i)}(i=1,\ldots,r)(k=1,\ldots,r)$ are positive numbers, $a_j^{(i)},b_j^{(i)}(i=1,\ldots,r),a_{kj},b_{kj}(k=2,\ldots,r)$ are complex numbers and here $m^{(i)},n^{(i)},p^{(i)},q^{(i)}(i=1,\ldots,r),n_k,p_k,q_k(k=2,\ldots,r)$ are non-negative integers where $0\le m^{(i)}\le q^{(i)},0\le n^{(i)}\le p^{(i)},q^k\ge 0,0\le n_k\le p_k$. Here (i) denotes the numbers of dashes. The contours L_i in the complex s_i -plane is of the Mellin-Barnes type which runs from $-w\infty$ to $+w\infty$ with indentations, if necessary, to ensure that all the poles of $\Gamma\left(b_j^{(i)}-\beta_j^{(i)}s_i\right)\left(j=1,\ldots,m^{(i)}\right)$ are separated from those of $\Gamma\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i\right)\left(j=1,\ldots,n_r\right)$.

For further details and asymptotic expansion of the I -function one can refer by Prasad [4].

In what follows, the multivariable I -function defined by [4] will be represented in the contracted notation:

$I^{0,n_2:\ldots:0,n_r:(m',n'):\ldots:(m^{(r)},n^{(r)})}_{p_2,q_2:\ldots:p_r,q_r:(p',q'):\ldots:(p^{(r)},q^{(r)})}\left[z_1,\ldots,z_r\right]$ Or simply by $I\left[z_1,\ldots z_r\right]$.

According to the asymptotic expansion of the gamma function, the counter integral (1.1) is absolutely convergent provided that

$$|\arg z_i|<\frac{1}{2}\pi U_i, U_i>0 \;;\; \mathrm{i}=1,\ 2,\ \ldots,\mathrm{r} \quad (1.4)$$

Where

$$U_i=\sum_{j=1}^{n^i}\alpha_j^{(i)}-\sum_{j=n^{(i)}+1}^{p^{(i)}}\alpha_j^{(i)}+\sum_{j=1}^{m^i}\beta_j^{(i)}-\sum_{j=m^{(i)}+1}^{q^{(i)}}\beta_j^{(i)}$$
$$+(\sum_{j=1}^{n_2}\alpha_{2j}^{(i)}-\sum_{j=n_2+1}^{p_2}\alpha_{2j}^{(i)})+(\sum_{j=1}^{n_3}\alpha_{3j}^{(i)}-\sum_{j=n_3+1}^{p_3}\alpha_{3j}^{(i)})$$
$$+\ldots+(\sum_{j=1}^{n_r}\alpha_{rj}^{(i)}-\sum_{j=n_r+1}^{p_r}\alpha_{rj}^{(i)})\quad-(\sum_{j=1}^{q_2}\beta_{2j}^{(i)}+\sum_{j=1}^{q_3}\beta_{3j}^{(i)}+\ldots+\sum_{j=1}^{q_r}\beta_{rj}^{(i)}) \quad (1.5)$$

The asymptotic expansion of the I -function has been discussed by Prasad [3]. His results run as follow:

$$I[z_1,\ldots,z_r]=0(|z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}),\max\{|z_1|,\ldots,|z_r|\}\to 0$$

Where

$$\alpha_i=\min\mathrm{Re}(b_j^{(i)}/\beta_j^{(i)}),\ j=1,\ \ldots,m^{(i)}\ ;\ \mathrm{i}=1,\ \ldots,\mathrm{r} \quad (1.6)$$

And $I[z_1,\ldots,z_r]=0(|z_1|^{\beta_1}\ldots|z_r|^{\beta_r}),\min\{|z_1|,\ldots,|z_r|\}\to\infty$

Where $\beta_i = \max \mathrm{Re}(\frac{a_j^{(i)}-1}{\alpha_j^{(i)}})$; $j=1, \ . \ . \ . \ . \ , n^{(i)}$, $\mathrm{i}=1, \ . \ . \ . \ . \ , \mathrm{r}$

The details of the function can be found in the paper of Prasad [4].

We shall give below three-term contiguous relation for the multivariable I -function and use them later on.

$$\text{(i)} \left[\Gamma\left(1-a_1+r+\frac{b_1}{\beta}\right)\right]^{-1} I^{0,n_2:\dots:0,n_r+1:(m',n'):\dots:(m^{(r)},n^{(r)})}_{p_2,q_2:\dots:p_r+1,q_r+1:(p',q'):\dots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\dots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\dots:(a_{rj},\alpha'_{rj},\dots,\alpha^{(r)}_{rj})_{1,p_r},(a_1+1-r;h',\dots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\dots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\dots:(b_{rj},\beta'_{rj},\dots,\beta^{(r)}_{rj})_{1,q_r},(b_1+1;\beta_{h'},\dots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\dots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right.\right]$$

$$= \beta\left[\Gamma\left(1-a_1+r+\frac{b_1}{\beta}\right)\right]^{-1} I^{0,n_2:\dots:0,n_r+1:(m',n'):\dots:(m^{(r)},n^{(r)})}_{p_2,q_2:\dots:p_r+1,q_r+1:(p',q'):\dots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\dots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\dots:(a_{rj},\alpha'_{rj},\dots,\alpha^{(r)}_{rj})_{1,p_r},(a_1-r;h',\dots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\dots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\dots:(b_{rj},\beta'_{rj},\dots,\beta^{(r)}_{rj})_{1,q_r},(b_1;\beta_{h'},\dots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\dots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right.\right]$$

$$- \beta\left[\Gamma\left(-a_1+r+\frac{b_1}{\beta}\right)\right]^{-1} I^{0,n_2:\dots:0,n_r+1:(m',n'):\dots:(m^{(r)},n^{(r)})}_{p_2,q_2:\dots:p_r+1,q_r+1:(p',q'):\dots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\dots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\dots:(a_{rj},\alpha'_{rj},\dots,\alpha^{(r)}_{rj})_{1,p_r},(a_1-r+1;h',\dots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\dots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\dots:(b_{rj},\beta'_{rj},\dots,\beta^{(r)}_{rj})_{1,q_r},(b_1;\beta_{h'},\dots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\dots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right.\right] \quad (1.7)$$

$$\text{(ii)} \ \beta(-1)^r\left[\Gamma\left(b_1+r-a_1\beta\right)\right]^{-1} I^{0,n_2:\dots:0,n_r+1:(m',n'):\dots:(m^{(r)},n^{(r)})}_{p_2,q_2:\dots:p_r+1,q_r+1:(p',q'):\dots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\dots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\dots:(a_{rj},\alpha'_{rj},\dots,\alpha^{(r)}_{rj})_{1,p_r},(a_1+1;h',\dots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\dots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\dots:(b_{rj},\beta'_{rj},\dots,\beta^{(r)}_{rj})_{1,q_r},(b_1+r;\beta_{h'},\dots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\dots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right.\right]$$

$$= (-1)^r\left[\Gamma\left(b_1+r-a_1\beta\right)\right]^{-1} I^{0,n_2:\dots:0,n_r+1:(m',n'):\dots:(m^{(r)},n^{(r)})}_{p_2,q_2:\dots:p_r+1,q_r+1:(p',q'):\dots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\dots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\dots:(a_{rj},\alpha'_{rj},\dots,\alpha^{(r)}_{rj})_{1,p_r},(a_1+1;h',\dots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\dots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\dots:(b_{rj},\beta'_{rj},\dots,\beta^{(r)}_{rj})_{1,q_r},(b_1+r;\beta_{h'},\dots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\dots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right.\right]$$

$$+ (-1)^r\left[\Gamma\left(b_1+r-1-a_1\beta\right)\right]^{-1} I^{0,n_2:\dots:0,n_r+1:(m',n'):\dots:(m^{(r)},n^{(r)})}_{p_2,q_2:\dots:p_r+1,q_r+1:(p',q'):\dots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\dots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\dots:(a_{rj},\alpha'_{rj},\dots,\alpha^{(r)}_{rj})_{1,p_r},(a_1+1;h',\dots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\dots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\dots:(b_{rj},\beta'_{rj},\dots,\beta^{(r)}_{rj})_{1,q_r},(b_1+r-1;\beta_{h'},\dots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\dots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right.\right] \quad (1.8)$$

$$\text{(iii)} \begin{vmatrix} r-b_1 & 1-a_1+r \\ \beta & 1 \end{vmatrix} I^{0,n_2:\dots:0,n_r+1:(m',n'):\dots:(m^{(r)},n^{(r)})}_{p_2,q_2:\dots:p_r+1,q_r+1:(p',q'):\dots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\dots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\dots:(a_{rj},\alpha'_{rj},\dots,\alpha^{(r)}_{rj})_{1,p_r},(a_1+1-r;h',\dots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\dots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\dots:(b_{rj},\beta'_{rj},\dots,\beta^{(r)}_{rj})_{1,q_r},(b_1+1;\beta_{h'},\dots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\dots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right.\right]$$

$$= I^{0,n_2:\dots:0,n_r+1:(m',n'):\dots:(m^{(r)},n^{(r)})}_{p_2,q_2:\dots:p_r+1,q_r+1:(p',q'):\dots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2} :\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1-r;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2} :\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1-r+1;\beta_{h'},\ldots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right]$$

$$- \beta I^{0,n_2:\ldots:0,n_r+1:(m',n'):\ldots:(m^{(r)},n^{(r)})}_{p_2,q_2:\ldots:p_r+1,q_r+1:(p',q'):\ldots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2} :\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1-r-1;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2} :\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1-r;\beta_{h'},\ldots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right] \quad (1.9)$$

(iv) $$(b_1-a_1+1) I^{0,n_2:\ldots:0,n_r+1:(m',n'):\ldots:(m^{(r)},n^{(r)})}_{p_2,q_2:\ldots:p_r+1,q_r+1:(p',q'):\ldots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2} :\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2} :\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1;\beta_{h'},\ldots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right]$$

$$= I^{0,n_2:\ldots:0,n_r+1:(m',n'):\ldots:(m^{(r)},n^{(r)})}_{p_2,q_2:\ldots:p_r+1,q_r+1:(p',q'):\ldots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2} :\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1-1;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2} :\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1;\beta_{h'},\ldots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right] \quad (1.10)$$

The contiguous relations (1.7), (1.8), (1.9) and (1.10) can be developed on lines similar to those given by Bushman and Gupta [1].

FINITE SUMMATION FORMULA

The finite summation formulae to be established are:

(i) $$\sum_{r=1}^{n} \left[\Gamma\left(1-a_1+r+\frac{b_1}{\beta}\right)\right]^{-1} I^{0,n_2:\ldots:0,n_r+1:(m',n'):\ldots:(m^{(r)},n^{(r)})}_{p_2,q_2:\ldots:p_r+1,q_r+1:(p',q'):\ldots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2} :\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1+1-r;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2} :\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1+1;\beta_{h'},\ldots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right]$$

$$= \beta \left[\Gamma\left(1-a_1+n+\frac{b_1}{\beta}\right)\right]^{-1} I^{0,n_2:\ldots:0,n_r+1:(m',n'):\ldots:(m^{(r)},n^{(r)})}_{p_2,q_2:\ldots:p_r+1,q_r+1:(p',q'):\ldots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2} :\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1-n;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2} :\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1;\beta_{h'},\ldots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right]$$

$$- \beta \left[\Gamma\left(-a_1+\frac{b_1}{\beta}\right)\right]^{-1} I^{0,n_2:\ldots:0,n_r+1:(m',n'):\ldots:(m^{(r)},n^{(r)})}_{p_2,q_2:\ldots:p_r+1,q_r+1:(p',q'):\ldots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2} :\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2} :\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1;\beta_{h'},\ldots,\beta_{h^{(r)}}):(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right] \quad (2.1)$$

(ii) $$\beta \sum_{r=1}^{n} (-1)^r \left[\Gamma\left(b_1+r-a_1\beta\right)\right]^{-1} I^{0,n_2:\ldots:0,n_r+1:(m',n'):\ldots:(m^{(r)},n^{(r)})}_{p_2,q_2:\ldots:p_r+1,q_r+1:(p',q'):\ldots:(p^{(r)},q^{(r)})}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1+r-1;\beta_{h'},\ldots,\beta_{h^{(r)}}:(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right]$$

$$=(-1)^n\left[\Gamma\left(b_1+n-a_1\beta\right)\right]^{-1} I^{0,n_2:\ldots:0,n_r+1:(m',n'):\ldots:\left(m^{(r)},n^{(r)}\right)}_{p_2,q_2:\ldots:p_r+1,q_r+1:(p',q'):\ldots:\left(p^{(r)},q^{(r)}\right)}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1+1;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1+n;\beta_{h'},\ldots,\beta_{h^{(r)}}:(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right]$$

$$-\left[\Gamma\left(b_1-a_1\beta\right)\right]^{-1} I^{0,n_2:\ldots:0,n_r+1:(m',n'):\ldots:\left(m^{(r)},n^{(r)}\right)}_{p_2,q_2:\ldots:p_r+1,q_r+1:(p',q'):\ldots:\left(p^{(r)},q^{(r)}\right)}$$

$$\left[z_1,\ldots,z_r \left| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\ldots:(a_{rj},\alpha'_{rj},\ldots,\alpha^{(r)}_{rj})_{1,p_r},(a_1+1;h',\ldots,h^{(r)}):(a'_j,\alpha'_j)_{1,p'}:\ldots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\ldots:(b_{rj},\beta'_{rj},\ldots,\beta^{(r)}_{rj})_{1,q_r},(b_1;\beta_{h'},\ldots,\beta_{h^{(r)}}:(b'_j,\beta'_j)_{1,q'}:\ldots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right. \right] \quad (2.2)$$

PROOF

To prove (2.1), putting $r=1,2,\ldots,n$ in (1.7)in succession and after taking the sum we see that in the resulting series on the right hand side the alternate terms cancel out, and we arrive at the required result (2.1).

Similarly, (2.2) and (2.3) can be established by using the results (1.8) and (1.9) respectively in place of (1.7). [Multiplying by the quantities $1,\beta^1,\beta^2,\ldots,\beta^n$ respectively only for (2.3)].

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