RELATIVISTIC MAGNETOHYDRODYNAMIC FLOWS IN KERR GEOMETRY

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ABSTRACT

During the infall, interactions between the magnetic field and the plasma are so strong that typically 10% of the rest mass energy is transported to magnetic field. If a small amplitude perturbation is assumed to superpose on the stationary black hole magnetosphere in which the magnetic field dominates the accretion, the former is likely to supply a lot of perturbation energy to the latter. Then, one may expect highly variable accretion inside the inner edge of the disk, it is favourable compared with the observation. Using these equations one may analyse perturbation of MHD accretion.

We present the basis equations for general relativistic magnetohydrodynamic (MHD) accretion onto a Kerr black hole. These equations may be use for the study of non-stationary and non-axsiymmetric perturbations of magnetohydrodynamic accretion onto a Kerr black hole. During the infall, interactions between the magnetic field and the plasma are so strong that typically 10% of the rest mass energy is transported to magnetic field. If a small amplitude perturbation is assumed to superpose on the stationary black hole magnetosphere in which the magnetic field dominates the accretion, the former is likely to supply a lot of perturbation energy to the latter.

Keywords: *Accretion, Black hole physics, galaxies, MHD.*

INTRODUCTION

Astrophysically it is very interesting to study of plasma accretion onto a black hole in the context of a galactic black hole candidate or an active galactic nucleus. Their output power is ultimately gravitational in origin the energy of their emission is supplied by accreting gas liberating its gravitational binding energy as shown by Rees (1984), Mitsuda et al (1984), Ubertini et al (1991), Motch, Ilovaisky and Chevalier (1985), Makino (1989), Rothschild et al (1983), Witta (1984), Covault et al (1992).

We have presented the basic equations for general relativistic magnetohydrodynamic flows in Kerr geometry. In a stationary black hole magnetosphere, the energy and the angular

momentum of infalling plasma are not conserved along a flow line as described by Hirotani et al (1992). Katkar et al (2009) again described it. We have also presented the critical conditions for general relativistic magnetohydrodynamic flows. These equations are very useful for the study of time variation of magnetohydro- dynamic accretion onto a black hole i.e. non-stationary and non-axisymmetric perturbations of

magnetohydrodynamic accretion onto a Kerr black hole.

BASIC EQUATIONS

Since the self-gravity of electromagnetic field and plasma around the black hole is very weak, the background geometry of the magnetospere is given by the Kerr metric

$$ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 + \frac{4Mar\sin^2\theta}{\Sigma} dt d\varphi - \frac{A\sin^2\theta}{\Sigma} d\varphi^2$$

$$-\frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2, \tag{1}$$

where

$$\Delta = r^2 - 2Mr + a^2, \tag{2}$$

$$\Sigma = r^2 + a^2 Cos^2\theta, \tag{3}$$

$$A = \left(r^2 + a^2\right) - \Delta a^2 \sin^2\theta, \tag{4}$$

$$a = J/M. \tag{5}$$

M be the mass of black hole and $C = G = 1.$

Under magnetohbydrodynamic conditions the electric field reduces to zero, hence, one obtains

$$F_{ik}U^i U^k = 0, \tag{6}$$

where F_{ik} be the electreonagnetic field tensor satisfying the Maxwell equations

$$F_{[k,l]} = 0, \tag{7}$$

where U^i be the fluid four velocity.

The motion of the fluid in the cold limit reads

$$T^{ik}_{;k} = \left[unU^iU^k + \frac{1}{4\pi}\left(F^{i\ell}F^k_\ell + \frac{1}{4}g^{ik}F_{mp}F^{mp} \right) \right]_{;k} = 0 \qquad (8)$$

where μ be the rest-mass of a proton and the semicolon denotes a covariant derivative. The proper number density n obeys the continuity equation

$$\left(nU^i \right)_{;i} = 0. \qquad (9)$$

These basic equations are used for the description of stationary and axisymmetric black hole magnetosphere.

Let us describe a stationary plasma accretion in a black hole magnetosphere. There are two light surfaces in a black hole magnetosphere. One as the outer light surface which is created by centrifugal force and other as the inner light surface which is formed by the gravity of the hole. In a region between the horizon and the inner light surface plasma must stream inwards, while in a region beyond the outer light surface it must stream outwards. The plasma source where both inflows and outflows start with low poloidal velocity will be located between two light surfaces as presented by Nitta et al (1991). The injection region $r = r_F$

successively, and at last reach the event horizon $r = r_H$. It is shown in fig.1.

CRITICAL CONDITIONS

Let us describe the critical conditions for general relativistic magnetohydrodynamic flows. There exist four integration constants conserved along each flow lines as shown by Bekenstein and Oran (1977) and Camenzind (1986a,b) from the analysis of stationary and axisymmetric magnetohydrodynamic equations. These conserved quantities are the angular velocity of a magnetic field line Ω_F, particle flux per magnetic flux tube η, total energy E, and the total angular momentum L.

$$\Omega_F = \frac{F_{tr}}{F_{r\varphi}} = \frac{F_{t\theta}}{F_{\theta\varphi}}, \qquad (10)$$

$$\eta = -\frac{\sqrt{-g}\,nU^r}{F_{\theta\varphi}} = -\frac{\sqrt{-g}\,nU^\theta}{F_{\varphi r}} = -\sqrt{-g}\,n\frac{\left(U^\varphi - \Omega_F U^t \right)}{F_{r\theta}}, \qquad (11)$$

$$E = \mu U_t - \frac{\Omega_F}{4\pi\eta}B_\varphi, \qquad (12)$$

$$L = -\mu U_\varphi - \frac{1}{4\pi\eta}B_\varphi, \qquad (13)$$

where the toroidal magnetic field B_φ reads

$$B_\varphi = -\frac{\rho_w^2}{\sqrt{-g}} F_{r\theta}, \tag{14}$$

where ρ_w^2 is given by

$$\rho_w^2 = g_{t\varphi}^2 - g_{tt} g_{\varphi\varphi} = \Delta \sin^2 \theta. \tag{15}$$

The poloidal flow line is similar with a poloidal magnetic field line and reads

$$\psi(r,\theta) = \text{constant.} \tag{16}$$

Fluid must pass through the critical points before they fall into the horizon. The poloidal wind equation as suggested by Camenzind (1986) reads

$$U_p^2 + 1 = \left(\frac{E}{\mu}\right)^2 \frac{KK_I (\mu/E)^2 - 2K_I (\mu/E)^2 M_A^2 - \bar{k}M_A^4}{\left(K - M_A^2\right)^2}, \tag{17}$$

where the poloidal velocity U_ρ and the Alfven Mach number M_A^2 are defined as

$$U_p^2 = -\left[g_{rr} \left(U^r\right)^2 + g_{\theta\theta} \left(U^\theta\right)^2 \right], \tag{18}$$

$$M_A^2 = \frac{4\pi\mu\eta^2}{n}, \tag{19}$$

and K, $\bar{K}$ read

$$K = g_{\varphi\varphi} \Omega_F^2 + 2g_{a\varphi} \Omega_F + g_{tt}, \tag{20}$$

$$\overline{K} = \frac{g_{\varphi\varphi} + 2g_{t\varphi}\left(L/E\right) + g_{tt}\left(L/E\right)^2}{\rho_w^2}, \tag{21}$$

Let us differentiate eq. (17) alone a poloidal flow line, we obtain the critical condition such that U'_p may not diverge

$$\left\{\frac{\left(\dfrac{K^3 B_p^2}{16\pi^2 \mu^2 \eta^2}\right)^{\frac{1}{3}} \left(K_I - K\right)^{\frac{1}{3}}}{K_I \left(\mu/E\right)^2 + K\overline{K}}\right\}' = 0, \tag{22}$$

$$U_p^2 = U_{FM}^2 = \frac{K_I - K}{M_A^2}, \tag{23}$$

where the prime $(')$ denotes the derivative $\partial_r - \left(\psi_r/\psi_\theta\right)\partial_\theta$, and the poloidal magnetic field as

$$\left(B_p\right)^2 = -g_{rr}\left(B^r\right)^2 - g_{\theta\theta}\left(B^\theta\right)^2$$

$$= -g_{rr}\left(-F_{\theta\varphi}/\sqrt{-g}\right)^2 - g_{\theta\theta}\left(-F_{\varphi r}/\sqrt{-g}\right)^2. \tag{24}$$

We define U_{FM} as the fast-magnetosonic velocity. Hirotani et al (1992) obtained the critical point as

$$\frac{r_F - r_H}{r_H} = -\frac{M\sum_H \Omega_F \left(\omega_H - \Omega_F\right)\sin^2\theta}{\left(r_H - M\right)\left(1 - a\Omega_F \sin^2\theta\right)^3 W^3}\frac{\mu}{E} \ll 1. \tag{25}$$

where

$$W^2 = 1 + \frac{2r_H(\omega_H - \Omega_F)\sin^2\theta}{(r_H - M)(1 - a\Omega_F \sin^2\theta)(1 - a\omega_H \sin^2\theta)}\big[a - F(\rho)\big], \quad (26)$$

$$F(p) = 2M^2(\omega_H - \Omega_F)\frac{(1 - a\omega_H \sin^2\theta)}{(1 - a\Omega_F \sin^2\theta)}\left[-\frac{\partial_\theta(P\sin\theta)}{\sin\theta}\right.$$

$$\left. +\left(1 - \frac{M}{r_H}\right)P^2 + \frac{4mr_H}{\sum_H}PCot\theta\right] \qquad (27)$$

$$P = -\left(r\psi_r / \xi_\theta\right) \qquad \text{(fined line shape)} \qquad (28)$$

$$\omega_H = a/2Mr_H \qquad \text{(Angular velocity of hole's rotation)} \quad (29)$$

But the smallness of factor

$$|\mu/E| \ll 1, \qquad (30)$$

ensures

$$\frac{(r_F - r_H)}{r_H} \ll 1. \qquad (31)$$

It is located near the horizon in the magnetically dominated limit.

CONCLUDING REMARKS

We present the basis equations for general relativistic magnetohydrodynamic (MHD) accretion onto a Kerr black hole. These equations may be use for the study of non-stationary and non-axsiymmetric perturbations of magnetohydrodynamic accretion onto a Kerr black hole. During the infall, interactions between the magnetic field and the plasma are so strong that typically 10% of the rest mass energy is transported to magnetic field. If a small amplitude perturbation is assumed to superpose on the stationary black hole magnetosphere in which the magnetic field

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