SPHERICALLY SYMMETRIC SPACETIMES

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ABSTRACT

We present few properties of spacetimes that are spherically symmetric and whose metrics obey the Einstein field equations with a perfect fluid source.

INTRODUCTION:

Misner and Sharp (1964) presented properties of spherically symmetric spacetimes although the idea was originated by Lemaitre (1933a). It was given also by Podurets (1964). Ruban (1983) investigated detailed interpretation of all the formulae. Misner (1965) generalised it to include null radiation. Often we discuss spherically symmetric solutions, so we present here few properties of spacetimes that are spherically symmetric and whose metrics obey the Einstein field equations with a perfect fluid source.

PROPERTIES OF SPHERICALLY SYMMETRIC PERFECT FLUID SOLUTIONS:

Let us consider the matric of the form

$$ds^2 = e^{\gamma(t,r)}dt^2 - e^{\alpha(t,r)}dr^2 - R^2(t,r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{1}$$

Let us suppose that the coordinates are comoving such that

$$u^\alpha = \bar{e}^{\gamma/2}\delta_0^\alpha . \tag{2}$$

Then in the course of integrating the Einstein field equations for (1) with perfect fluid source, one obtains

$$\left(\frac{4\pi}{c^2}\right)\rho R^2 R_{,r} = \frac{\partial m}{\partial r}, \tag{3}$$

$$\left(\frac{4\pi}{c^2}\right)pR^2 R_{,t} = \frac{-\partial m}{\partial t}, \tag{4}$$

where

$$m=\left(\frac{c^2}{2G}\right)\left[R+\bar{e}^{\gamma}RR_{,t}^2-\bar{e}^{\alpha}R,R_{,r}^2-\frac{1}{3}\Lambda h^3\right]. \quad (5)$$

with Λ as cosmological constant. The remaining field equations are

$$\gamma_{,r}=-2p_{,r}/(\rho+p) \quad (6)$$

$$\alpha_{,t}=-2\rho_{,t}/(\rho+p)-4\frac{R_{,t}}{R} \quad (7)$$

In view of eq. (3), the function $m(t,r)$ is recognised as the mass equivalent to the total energy contained within the comoving shell of radial coordinate r at the time t. For $p=0,$ the eq. (4) implies that the total mass of such a body is conserved during evolution. For dust, when $p=0$, the mass is conserved within every shell r = constant.

For $R_{,r}=0,$ very interesting thing happens. The metric (1) is then an spherically symmetric, but does not contain the centres of symmetry in the hypersurfaces t = constant. In Russian literature, such solutions are known as T-models as presented by Novikov (1963), Ruban (1968), (1969) and (1983). For them, the standard Schwarzschild coordinates in which *R = r* do not exist. Then, in view of eqs. (3) and (4), one obtains

$$\frac{\partial m}{\partial r}=0. \quad (8)$$

Now *m(t)* is obtained by its initial value, the same for all shells. If, in addition, *p = 0*, then m = constant. Ruban (1968, 1969, and 1983) presented that this may be interpreted as follows:

In a T-model of dust, any amount of mass added to the source has its effect on the gravitational field exactly cancelled by its gravitational mass defect, so that the active gravitational mass remains the same for all shells and all time when p = 0, then $\gamma=\gamma(t)$ from eq. (6), and so $\gamma=0$ may be obtained by a transformation of t. When $R_{,r}=0$ in addition, then the eq. (5) assumes the form

$$R_{,t}^2=-1+2Gm/c^2R+\frac{1}{3}\Lambda R^2, \quad (9)$$

and this is the equation of motion of any shell r = constant. It is to be noted that with $R_{,r}=0,$ eqs. (3) - (7) are not sufficient to obtain in solution, eq. (3) does not evaluate ρ then, and eq. (7) only gives a connection between α and ρ. This is because eq. (3) is obtained from the $(0,0)$ field equation by multiplying it by $R^2R_{,r}$, and so becomes an identity when $R_{,r} = 0.$ In this case, in order to obtain a solution, one has to go back to Einstein field equations in their raw form. The vaccum T-model is

$$ds^2 = \left(\frac{2Gm}{c^2 t} - 1\right)^{-1} dt^2 - \left(\frac{2Gm}{c^2 t} - 1\right) dr^2 - t^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \qquad (10)$$

CONCLUDING REMARKS

We have investigated some important properties of spacetimes that are spherically symmetric and whose metrics obey the Einstein field equations with a perfect fluid.

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