DISCUSSION : ON η-VACUUM IN QUANTUM GRAVITY

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ABSTRACT

We present a new solution to the cosmological constant problem in quantum theory of gravity with torsional instantons. These particles exist in a first order formulation of Giddings-Strominger axionic gravity carrying Nich-Yan topological charge. Due to tunneling effects, the nonperturbative ground state is produced and it is shown to be stable under quantum fluctuations. It is also shown that in view of the Hubble constant, the associated vacuum angle known as Barbero-Immirzi topological parameter, gets fixed to a numerical value. The energy of the η -vacuum has a natural interpretation as the dark energy which is believed to be responsible for the accelerated expansion of the universe at present. We have presented a new solution to dark energy or to the cosmological constant problem in a nonperturbative quantum theory of gravity with torsional instantons. In this framework, the Barbero-Immirzi topological parameter gets fixed to a value close to $\pi/2$ radians, provided the ground state energy receives no other contributions from any other vacuum angle in quantum theory.

Keywords: *Barbero-Immirzi topological parameter, Nich-Yan topological charge, η-vacuum, torsional instantons.*

INTRODUCTION

Eguchi and Freund (1976), Backler and Hehl (2011), Nieh and Yan (1982), Neih (2007), Date, Kaul and Sengupta (2009), Kaul and Sengupta (2012), Sengupta (2012), Rezende and Perez (2009) have presented to include three topological densities in the Lagrangian known as the Euler, Pontryagin and Nieh-Yan. These topological terms may be expected in quantum theory of gravity via potential instantonic effects in the Euclidean formulation as shown by Coleman (1979). Such expectations have found by Hawking (1977), Eguchi and Hanson (1979), Eguchi et al (1980), Gibbons and Hawking (1979), Smilga (1984) with the discovery of gravitational instantons carrying pontryagin and Euler charges. Kaul and Sengupta (2014) presented an example for a Neih-Yan instanton which may solve the classical equations of motion of Euclidean gravity with or without matter. In the work of Kaul and Sengupta, it was presented that the Giddings-Strominger wormhole are the solutions of a second order theory of axionic gravity, which may be interpreted as torsional preudoparticles carrying nontrivial Nieh-

Yan topological numbers. The vacuum angle η associated with this quantum gravity was identified as Barbero-Immirzi parameter as given by Fernando and Barbero (1995), Immirzi (1997). Sengupta (2013, 2010) has shown that in the effective Lagrangian it appears as the coupling constant multiplying the Nieh-Yan topological density. For the first time, Kaul and Sengupta (2014) have shown that there exists similarity between gauge theories with vacuum and gravity theory with η vacuum.

Coleman (1988) has used wormhole physics to solve the cosmological constant problem and have been discussed by many other workers such as Hawking (1984), Linde (1988), Klebanow et al (1989), Hawking (1990). In the context of SU(2) gauge theories Yokoyama (2002) has discussed the cosmological constant problem. The cosmological constant problem has also been presented from various perspectives by Weinberg (1989), Copenland et al (2006) and Padmanabhan and Padmanabhan (2014).

THE TORSIONAL INSTANTON AND η -VACUUM IN QUANTUM GRAVITY

In view of Kaul and Sengupta (2014), one may write down the first order Lagrangian density for axionic gravity

$$L(e,\omega,\beta) = -\frac{1}{2k^2} e e_I^{\mu} e_J^{\nu} R_{\mu\nu}^{IJ}(\omega) + \frac{1}{2K} e H^{\mu\nu\alpha} e_{\mu}^{I} D_{\nu}(\omega) e_{\alpha I}$$

$$+\beta e H^{\mu\nu\alpha} H_{\mu\nu\alpha} \tag{1}$$

where $\beta_{\mu\nu}$ as the antisymmetric tensor and

$$R_{\mu\nu}^{IJ}(\omega) = \partial_{[\mu}\omega_{\nu]} + \omega_{[\mu}^{IL}\omega_{\nu]L}^{J}, \tag{2}$$

$$D_{\mu}(\omega) e_{\mu}^{J} = \partial_{\mu} e_{\mu}^{I} + \omega_{\mu}^{IJ} e_{\nu J} \tag{3}$$

$$H_{\mu\nu\alpha} = \partial_{[\mu}\beta_{\nu\alpha]}. \tag{4}$$

Let us varry eq. (1) with respect to ω_{μ}^{IJ}, one obtains nonvanishing torsion

$$T^{I}_{\alpha\beta} \equiv \frac{1}{2} D_{[\alpha}(\omega) e^{I}_{\beta]} = -\frac{k}{2} e^{\mu I} H_{\alpha\beta\mu} \tag{5}$$

Now let us put

$$\omega^{IJ}_{\mu} = \omega^{IJ}_{\mu}(e) + k^{IJ}_{\mu}, \tag{6}$$

where first part gives torsionless part and second as contortion in the right hand side of eq. (6), and the identity

$$k_{\mu\nu\alpha} = k^{IJ}_{\mu} e_{\nu I} e_{\alpha J} = T_{\mu\alpha\nu} - T_{\nu\mu\alpha} - T_{\alpha\mu\nu}. \tag{7}$$

Hence, one obtains contortion

$$k_{\mu\nu\alpha} = \frac{k}{2} H_{\mu\nu\alpha} \tag{8}$$

Let us now put the Giddings-Strominger ansatz as

$$ds^2 = d\tau^2 + a^2(\tau)\left[d\chi^2 + \sin^2\chi d\theta^2 + \sin^2\chi \sin^2\theta d\varphi^2\right] \tag{9}$$

$$H^{\tau ab} = 0 \tag{10}$$

$$H^{abc} = \frac{1}{\sqrt{g}} \in^{abc} h(\tau, \chi, \theta, \varphi), \tag{11}$$

where $h(\tau, \chi, \theta, \varphi)$ as a scalar function and $\in^{\tau abc} = \in^{abc}$ is a totally antisymmetric density on the three-sphere and whose indices are lowered by three-metric g_{ab}. One may solve for $\beta_{\mu\nu}$ to obtain

$$h(\tau) = \frac{KQ}{3a^3(\tau)} , \tag{12}$$

where Q be the axion charge defined as

$$\int d^3x \in^{abc} H_{abc} = 2\pi^2 KQ. \tag{13}$$

An evolution equation for $a(\tau)$ describes the Giddings-Strominger wormhole situation is

$$\dot{a}^2(\tau) = 1 - \frac{K^4 F_a^2 Q^2}{18a^4(\tau)}, \tag{14}$$

where

$$F_a^2 = \beta - \frac{1}{8}, \tag{15}$$

with

$$a_0 = 18^{-1/4} K (F_\alpha Q)^{\frac{1}{2}} , \tag{16}$$

representing as a throat of radius a_0 and a tunneling between two R^3 geometries separated in time. The half wormhole tunnels between R^3 and R^3+5^3 as instanton.

Now Nieh-Yan topological charge comes out as Q which is the axion charge

$$N_{NY} = \frac{1}{2\pi^2 K} \int_{5^3} d^3x \in^{abc} H_{abc} = Q. \tag{17}$$

Hence, there are an infinite number degenerate ground states in gravity theory and each of them is given by a definite Nieh-Yan number N_{NY} and each half-wormhole has a action

$$S = \frac{\pi^3}{\sqrt{2}} F_a Q = 3\pi^3 \frac{a_0^2}{K^2}. \tag{18}$$

Thus, there exists an amplitude to tunnel between any two states of N_{NY} numbers. As a result a nonperturbative η vacuum comes in picture due to superposition of all the perturbative ground states and one obtains the Barbero-Immirzi parameter η as a vacuum angle

$$\left|\eta\right\rangle = \sum_{N_{NY}} e^{i\eta N_{NY}} \left|N_{NY}\right\rangle. \tag{19}$$

The transtion amplitude in η -vacuum reads

$$\left\langle \eta' \right| \overline{e}^{FT} \left|\eta\right\rangle = A\delta(\eta-\eta')\exp\left[2\overline{e}^{S} KVTCos\eta\right], \tag{20}$$

where K denotes the contribution from quantum fluctuations. This presents that there is correction to the vacuum density

$$\frac{F_\eta}{V} = -2\overline{e}^{S} KCos\eta. \tag{21}$$

It is an important to note that there may be additional vacuum angle i.e. the coefficient of the Pontryagin density in the Lagrangian as

$$N_p = \int d^4 x e^{\mu\nu\alpha\beta} R_{\mu\nu}^{IJ} R_{\alpha\beta IJ}. \tag{22}$$

But for the canonical choice of the axion coupling one obtains $\beta = \perp$ or $F_a^2 = \frac{1}{24}$, the Pontryagin number is zero. Let us assume that there is only Barbero-Immirzi angle η in the quantum theory thereby the wormhole size a_o reads

$$a_0 = \frac{K}{2.3^{3/4}}. \tag{23}$$

VACUUM ENERGY DENSITY

In view of equations of motion for $\beta_{\mu\nu}$ and contortion $k_{\mu\nu\alpha}$, the eq. (1) gives the action

$$S(a)=\frac{6\pi^2 a_o^2}{k^2}\int d\tau\left[-\left(1+a^2(\tau)\right)a(\tau)+\frac{1}{a^3(\tau)}\right], \tag{24}$$

where

$$\tau\to\frac{\tau}{a_0},\qquad a(\tau)=\frac{a(\tau)}{a_0}. \tag{25}$$

Thus, we get

$$S^2S(a)=\frac{6\pi^2 a_0^2}{K^2}\int d\tau\delta a(\tau)\left[a(\tau)\frac{d^2}{d\tau^2}+\dot{a}(\tau)\frac{d}{d\tau}+\frac{8}{a^5(\tau)}\right]\delta a(\tau) \tag{26}$$

Now let us attempt to obtain the eigenvalues of the operator $\breve{\theta}$

$$\breve{\theta}=a(\tau)\left[a(\tau)\frac{d^2}{d\tau^2}+\dot{a}(\tau)\frac{d}{d\tau}+\frac{8}{a^5(\tau)}\right], \tag{27}$$

by using

$$du=\frac{d\tau}{a(\tau)} \tag{28}$$

Hence, one may obtain the one-instanton contribution as

$$A\overline{e}^{S}\int\frac{a_0}{K}d[\delta a]\overline{e}^{3\pi^2 a_0^2/K^2\int d\tau da(\tau)\breve{\theta}da(\tau)}$$

$$=A\overline{e}^{S}kVT, \tag{29}$$

where the spacetime volume VT comes due to an integration over the instanton in the four geometry. Let us now redefine the scale factor $a(\tau)$

$$\chi^2(\tau)=1-\frac{1}{a^4(\tau)}. \tag{30}$$

Hence, the eigenvalue equation for $\hat{\theta}$ assumes the form

$$4\left(1-\chi^2\right)\frac{d}{d\chi}\left[\left(1-\chi^2\right)\frac{d\psi}{d\chi}\right]+8\left(1-\chi^2\right)\psi=\lambda\psi. \tag{31}$$

This, one may obtain the only eigenmode which is finite at boundaries as

$$\psi\left(\chi\right)=\sqrt{2}\left(1-\chi^2\right)^{1/2}, \tag{32}$$

which corresponds to a positive eigenvalue

$$\lambda=4, \tag{33}$$

and is normalised with

$$\int du\left|\psi\right|^2=1. \tag{34}$$

One may obtain other solutions as

$$\psi=\left(1-\chi^2\right)^{-1},\chi\left(1-\chi^2\right)^{-1/2},\chi\left(1-\chi^2\right)^{-7/2}, \tag{35}$$

and these all diverge at boundaries.

Due to the time translation invariance there is also a zero mode among the fluctuations and reads

$$\psi_0=\dot{a}\left(\tau\right), \tag{36}$$

up to normalisation. As this eigenfunction does not vanish at the R^3 boundary, its normalisation may be defined after a regularisation. Hence, we get

$$\int_{reg} du\left|\psi_0\right|^2=\int du\left|\frac{da}{du}\right|^2+\frac{1}{2}a^2\dot{a}\Big|_{R^3}=\frac{\pi}{4}, \tag{37}$$

which is equal to the dimensionless instanton action. As a result, one obtains

$$k=\left[\frac{\sqrt{\pi}}{2}\frac{a_0}{K}\frac{k}{\sqrt{3\pi a_o}}\right]\left(Det\hat{\theta}\right)^{-1/2}=\frac{1}{4\sqrt{3\pi}}. \tag{38}$$

The three order zero modes, corresponding to the spatial coordinates (χ,θ,φ) are equivalent to an integration over the location over the three volume. In view of eq. (38) one may obtain the exact vacuum energy density in terms of only vacuum angle η

$$\langle\eta|\left[\frac{F_\eta}{V}\right]|\eta\rangle = -\frac{1}{2\sqrt{3\pi}}\bar{e}^{\pi^3/4\sqrt{3}} Cos\eta. \quad (39)$$

The above expression is an exact correction to the vacuum energy density in terms of only one free parameter i.e. vacuum angle η due to tunneling.

F_η AS DARK ENERGY OR THE EFFECTIVE COSMOLOGICAL CONSTANT

The instanton density η reads, in view of above equations of k and S,

$$\eta \sim 10^{-2} k^{-4}, \quad (40)$$

showing that the average separation between instantons is of the order of $10^2 K^4$, which is larger than the size of an instantion as

$$a_0^4 \sim 10^{-3} K^4. \quad (41)$$

In view of Lorentz invariance, the effective energy momentum tensor reads

$$\langle\eta|T_{\mu\nu}|\eta\rangle = -\frac{F_\eta}{V} g_{\mu\nu}. \quad (42)$$

Hence, one may identify F_η as the dark energy or the effective cosmological constant. The current observations favour the presence of a tiny amount of vacuum energy which is thought to be driving accelerated expansion of the universe.

Let us suppose that our universe has reached a stage where it is in true η -vacuum state of quantum gravity. Hence, eq. (3.9) must be equal to the energy density ρ of the present universe. But ρ is about 70% of ρ_c i.e. critical density

$$\rho_c = \frac{3H_0^2}{4\pi G} \quad (43)$$

where H_0 be the Hubble constant.

Since the energy density given by eq. (39) must be positive, the vacuum angle η reads

$$\frac{\pi}{2} \leq \eta \geq \frac{3\pi}{2}. \tag{44}$$

Thus, we obtain

$$-2\bar{e}^{S} kCos\eta = \alpha \bar{e}^{276}, \tag{45}$$

or

$$\ln(-Cos\eta) = S - 276 + \ln\left(\frac{\alpha}{2k}\right) \tag{46}$$

But $\alpha \approx 0.7,$ and one obtains

$$\ln(-Cos\eta) = -269.7 \tag{47}$$

or

$$\eta = \frac{\pi}{2} + \sin^{-1}\left(\bar{e}^{269.7}\right) \tag{48}$$

$$= \frac{\pi}{2} + \delta(H_0). \tag{49}$$

In view of the above, the Barbero-Immirzi parameter η is fixed for the value of $\frac{\pi}{2}$ radians. The $\delta(H_0)$ is obtained in terms of current value of Hubble parameters. The effective energy momentum tensor reads

$$\langle T_{\mu\nu} \rangle = -\sum_i \frac{F_{\theta_i}}{V} g_{\mu\nu}. \tag{50}$$

CONCLUDING REMARKS

Due to tunneling effects, the non perturbative ground state is produced and it is shown to be stable under quantum fluctuations. It is also shown that in view of the Hubble constant, the associated vacuum angle known as Barbero-Immirzi topological parameter, gets fixed to a numerical value. The energy of the η -vacuum has a natural interpretation as the dark energy which is believed to be responsible for the accelerated expansion of the universe at present. We have presented a new solution to dark energy or to the cosmological constant problem in a nonperturbative quantum theory of gravity with torsional instantons. In this framework, the Barbero-Immirzi topological parameter gets fixed to a value close to $\pi/2$ radians, provided the ground state energy receives no other contributions from any other vacuum angle in quantum theory.

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