HIGHER DIMENSIIONAL REISSNER-NORDSTROM BLACK HOLES

Amit Kumar Srivastava,

*Department of Physics, D.A.V. College,
Kanpur (U.P.), India*

ABSTRACT

We present higher dimensional Reissner-Nordstrom black holes. We also recover the Schwarzschild and Schwarzschild-de Sitter solutions.

INTRODUCTION

Salgado (2003) proved a theorem characterising static spherically symmetric solutions to the Einstein's field equations in four dimensions by imposing certain conditions on the energy momentum tensor. This theorem gives exact solutions like black holes with different energy momentum tensors. Gallo (2004) extended the theorem and obtained a method to generate static black holes in higher dimensional spacetimes. Radiating Kerr-Newman black holes in f(r) gravity has been presented by Ghosh, Maharaj and Papnoi (2013). Datta and Bose (2019) have presented quasi-normal modes of static spherically symmetric black holes.

In this work we have presented Reissner-Nordstrom (DS/ADS) black hole solutions in higher dimensions without imposing any condition on energy momentum tensor.

THE METRIC AND SOLUTIONS

Let us consider the most general static and spherically symmetric metric in D-dimensions

$$ds^2 = -B^2(r)dt^2 + A^2(r)dr^2 + r^2 d\Omega_{D-2}^2, \quad (1)$$

where

$$d\Omega_{D-2}^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \ldots.. + \prod_{n=1}^{D-3}\sin^2\theta_n d\theta_{D-2}^2. \quad (2)$$

Let us write the Einstein-Maxwell equations in D-dimensions with a cosmological constant Λ. So, in view of Einstein-Maxwell action in D-dimensions

$$S = \int d^D x\sqrt{|g|}\left[R - 2\Lambda + \frac{K}{8\pi}F_{\alpha\beta}F^{\alpha\beta}\right], \quad (3)$$

where

$$K = 8\pi G / c^4 \tag{4}$$

$$F_{\alpha\beta} = A_{\alpha;\beta} - A_{\beta;\alpha}$$

One obtains the following Einstein-Maxwell equations

$$R_{\alpha\beta} - \frac{1}{2} q_{\alpha\beta} R + \Lambda g_{\alpha\beta} = \frac{K}{4\pi}\left[F_{\alpha}^{\gamma} F_{\beta\gamma} - \frac{1}{4\pi} q_{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta}\right], \tag{5}$$

$$F_{\alpha;\gamma}^{\gamma} = 0, \tag{6}$$

$$F_{\alpha\beta;\gamma} + F_{\beta\gamma;\alpha} + F_{\gamma\alpha;\beta} = 0, \tag{7}$$

and $G = c = 1.$

Hence, the only nontrivial components from $F_{\alpha\beta}$ are

$$F_{tr} = -F_{rt} = \frac{Q}{r^{D-3}}, \tag{8}$$

where Q stands for an isolated point charge.

By solving the Einstein-Maxwell equations for the metric of eq. (1), one finds for D > 3

$$B^2(r) = \frac{1}{A^2(r)}$$

$$= 1 - \frac{2M}{r^{D-3}} + \frac{2Q^2}{(D-3)(D-2)r^{2(D-3)}} - \frac{2\Lambda r^2}{(D-2)(D-1)}, \tag{9}$$

where M be the constant of integration.

It is obvious that this metric is a generalisation to D-dimensions of Reissner-Nordstrom (RN)-dS/AdS according to the sign of $\Lambda.$

For Q = 0, $\Lambda > 0,$ we obtain the Schwarzschild-de Sitter (dS) black holes in D-dimensions. Again for D = 4, one obtains

$$B(r) = A^{-1}(r) = \left[1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right]^{\frac{1}{2}}, \qquad (10)$$

which is well known Reissner-Nordstrom metric in four dimensions.

For D = 3, we obtain the metric for a charged de Sitter/Anti de Sitter (dS/AdS) black hole in three dimensions

$$ds^2 = -\left[\left(\bar{M} - \Lambda r^2 - 2Q^2 \ln r\right)\right] dt^2 + \left[\left(\bar{M} - \Lambda r^2 - 2Q^2 \ln r\right)\right]^{-1} dr^2$$

$$+ r^2 d\theta_1^2, \qquad (11)$$

where

$$\bar{M} = (1 - 2M).$$

CONCLUDING REMARKS

We have presented D-dimensional Reissner-Nordstrom (dS/AdS) black holes. In this way for Q = 0, $\Lambda > 0$, we investigated to Schwarzschild-de Sitter black holes in D-dimensions and for D = 4 we have recovered Reissner-Nordstrom metric in four dimensions. Again for D = 3, we obtained a metric for charged de Sitter/Anti de Sitter (dS/AdS) black holes in three dimensions.

REFERENCES

- Salgado, M. (2003), Class. Quant. Grav. 20, 4451.
- Gallo, E (2004), G.R.G. Vo. 36, no. 6.
- Ghosh, S.G et al (2013), ar Xiv: 1208.3028 v2 (gr-qc)
- Datta, S. and Bose, S. (2019) IUCAA Pune.
- Berry, C.P.L. and Gair, J.R. (2011) Phys. Rev. D-83, 104022.