STATIC SPHERICALLY SYMMETRIC BLACK HOLES IN f(R) GRAVITY

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ABSTRACT

We present basic equations for f(R) gravity and study static spherically symmetric black holes in f(R) theory in D-dimensional spacetime. We recover de Sitter and anti de Sitter solutions (dS/AdS).

INTRODUCTION

The f(R) gravity comes in picture due to Felice and Tsujikawa (2010), Nojiri and Odinatsov (2007, 2011) and Capozziello (2011) and other workers, as straightforward extension of general theory of relativity. In such theories the scalar curvature R in the Einstein-Hilbert action is replaced by R+f(R). It is very simple alternative to general theory of relativity. Abbott et al (2016, 2017) opened a new phase in the history of physics. Very little is known about f(R) gravity exact solutions, which deserve to be understood better. Spherically symmetric black hole solutions were presented for a positive constant curvature scalar and a black hole solutions were also obtained in f(R) gravities in the case of negative constant curvature scalar by different workers. Here we present some static spherically symmetric black hole solutions in f(R) gravity in D-dimensional spacetimes.

GENERAL EQUATIONS FOR f(R) GRAVITY

Let us start by considering a Lagrangian of the form

$$S_f=\frac{1}{2k^2}\int d^4x\sqrt{-g}\,f(R), \quad (1)$$

where $k^2=8\pi.$ Varrying this action with respect to metric $g_{\alpha\beta}$ gives the equation of motion

$$R_{\alpha\beta}f'(R)-\frac{1}{2}f(R)g_{\alpha\beta}+\left(g_{\alpha\beta}\Box-\nabla_\alpha\nabla_\beta\right)f'(R)=0. \quad (2)$$

It is well known that eqs. (2) has a constant curvature solution $R=\bar{R},$ so we get

$$\bar{R}_{\alpha\beta}f'(R)-\frac{1}{2}g_{\alpha\beta}f\left(\bar{R}\right)=0. \quad (3)$$

Its trace reads

$$\bar{R}f'\left(\bar{R}\right)-2f\left(\bar{R}\right)=0. \quad (4)$$

Let us express the Lagrangian as

$$f(R)=R+\varphi(R), \quad (5)$$

where $\varphi(R)$ represents terms in the gravity action beyond general theory of gravity. Therefore, we rename $\varphi(R)$ as $f(R)$. Thus, Lagrangian assumes the form $R+f(R)$.

GRAVITY IN HIGHER DIMENSIONAL SPACETIME

Let us consider the modified Einstein-Hilbert, D-dimensional action

$$S=\frac{1}{16\pi}\int d^D x\sqrt{-g}\left(R+f(R)\right), \quad (6)$$

where g be the determinant of the metric $g_{\alpha\beta}$ $(\alpha,\beta=0,1........D-1)$, R be the scalar curvature and $f(R)$ as the real function. Here, it is to noted that the Einstein-Hilbert action in D-dimension for Λ_f as cosmological constant reads

$$f(R)=-(D-2)\Lambda_f. \quad (7)$$

In view of eq. (6), the equation of motion is given by

$$R_{\alpha\beta}\left(1+f'(R)\right)-\frac{1}{2}\left(R+f(R)\right)g_{\alpha\beta}$$

$$+\left(g_{\alpha\beta}\nabla^2-\nabla_\alpha\nabla_\beta\right)f'(R)=2T_{\alpha\beta}, \quad (8)$$

where $R_{\alpha\beta}$ as usual Ricci tensor, the prime in $f'(R)$ gives differentiation with respect to R, and $\nabla^2=\nabla_\alpha\nabla^\alpha$ alongwith ∇ as the usual covariant derivative. Let us introduce $R=R_0$ as constant scalar curvature.

The trace of eq. (8) gives

$$2\left(1+f'(R_0)\right)R_0-D\left(R_0+f(R_0)\right)=0, \quad (9)$$

where

$$T=T_\alpha^\alpha=0, \quad (10)$$

$$g_\alpha^\alpha=\delta_\alpha^\alpha=D. \quad (11)$$

Eq. (9) gives the negative constant scalar curvature as

$$R_0=\frac{Df(R_0)}{2\left(1+f'(R_0)\right)-D}=D\Lambda_f, \quad (12)$$

(6)

$$\Lambda_f=\frac{R_0}{D}, \quad (13)$$

where Λ_f as an effective cosmological constant. Therefore, for any $R=R_0$ such that

$$1+f'(R_0)\neq 0, \quad (14)$$

must obeys (7)

$$R_{\alpha\beta}=\Lambda_f g_{\alpha\beta}+\frac{2}{1+f'(R_0)}T_{\alpha\beta}. \quad (15)$$

Again for conformal matter or $T=T_\alpha^\alpha=0$ with $\Lambda_f\neq 0$, one has again constant $R=R_0$ with $R_0=D\Lambda_f$ and $g_{\alpha\beta}$ be the (8) solution of f(R) provided that

$$f\left(D\Lambda_D\right)=\Lambda_D\left(2-D+2f'\,D\Lambda_D\right) \quad (16)$$

For this case the solution is de Sitter (dS) or anti de-Sitter (AdS) in accordance with sign of R_0 as in general relativity with cosmological constant.

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